94 Wallday

Maths III. Hockbram March 1955. Hard VA

i. $2\pi R^2 + 2\pi RH$

$= 2\pi R(R+H)$

$= 2 \times \frac{22}{7} \times \frac{25}{4} \left(6\frac{1}{4} + 5\frac{3}{4}\right)$

$= \frac{11}{\cancel{22}} \times \frac{25}{\cancel{2}} \times 12$

$= \frac{275 \times 12}{7}$

$= \frac{3300}{7}$

$= 471.438$

$= \underline{471.44}$ (to 2 dec places)

3

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ii. $v^2 = u^2 + 2fs$

$s = \frac{v^2 - u^2}{2f}$

$= \frac{289 - 9}{2.16}$

$= \frac{280}{\cancel{32}}$ (70, 8)

$= \underline{8\frac{3}{4}}$

3

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iii.

$AX^2 = AB.XB$

$x^2 = 10(10 - AB)$

$x^2 = 100 - 10AB$

x

3 2

$x^2 = 100 - 10x$

$x^2 + 10x - 100 = 0$

3

2i. Let £x be C.P.

catalogue price = £$\frac{125}{100}x$

S.P. = £$\frac{90}{100} \times \frac{125}{100} \times x$

But S.P. = £2¼.

$\therefore \frac{90}{100} \times \frac{125}{100} \times x = \frac{9}{4}$

$x = \frac{9}{4} \times \frac{100}{125} \times \frac{100}{90}$

cost price = £2.

4

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ii $4x + 7y = 15$ (1)

and $9x - 13y = 5$ (2)

$36x + 63y = 135$ (3)

$36x - 52y = 20$ (4)

$115y = 115$

$y = 1$

$18 - 13 = 5$. = R.H.S.

$4x = 8$

$x = 2$

4

3i. 1" = 2·54 cms H. Hash Maths III Mauch Ass. VR.

1 metre = 100 cms

1 km = 1000 metres

1 cm = $\frac{1}{2\cdot54}$ ins

1 metre = $\frac{100}{2\cdot54}$ ins

1 km = $\frac{100,000}{2\cdot54}$ ins

1 ins = $\frac{2\cdot54}{100,000}$ kms.

63,360 ins = $\frac{2\cdot54 \times 63,360}{100,000}$ kms.

1 mile = 1·61 kms (to 3 sig figs)

5 mls = 8·045 kms.

error = ·045 kms taking 5 mls = 8 kms.

% error = $\frac{\cdot045}{8\cdot045} \times 100$ %

$= \frac{4\cdot5}{8\cdot045}$

$= \frac{\cdot9}{1\cdot609}$

= ·5593 % error

= 0·6 % error

No	Log
100,000	5·00000
2·54	0·40548
63,360	4·80198
	5·20666
100,000	5·00000
1·609	0·20666
·9	T·95542
1·609	0·20666
·5593	T·74876

9

P.J. Marsh. Maths III March 1955. VA.

4 If $\frac{1}{(x+2)} + \frac{1}{(x-5)} = \frac{3}{4}$

then $4(x-5) + 4(x+2) = 3(x+2)(x-5)$

$4x - 20 + 4x + 8 = 3(x^2 - 3x - 10)$

$6x - 20 + 4x + 8 = 3x^2 - 9x - 30$

$4x + 4x - 3x^2 + 9x = -30 + 20 - 8$

$-3x^2 + 17x = -18$

$3x^2 - 17x = 18$

$x^2 - \frac{17x}{3} = 6$

$x^2 - \frac{17x}{3} + \left(\frac{17}{6}\right)^2 = 6 + \frac{289}{36}$

$\left(x - \frac{17}{6}\right)^2 = \frac{216}{36} + \frac{289}{36}$

$\left(x - \frac{17}{6}\right)^2 = \frac{505}{36}$

$x - \frac{17}{6} = \pm \frac{22.47}{6}$

$x = \frac{17}{6} + \frac{22.47}{6}$ or $x = \frac{17}{6} - \frac{22.47}{6}$

$x = \frac{39.47}{6}$ or $x = -\frac{5.47}{6}$

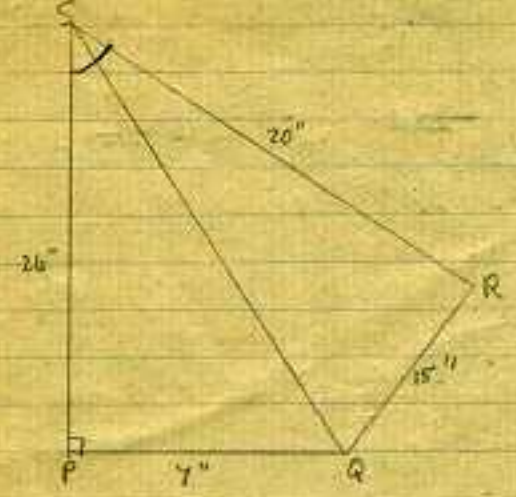
$x = 6.578$ or $x = -0.91$

$x = 6.58$ or $x = -0.91$ (to 2 dec places)

10

Good Sol'n

5.



By Pythagoras $QS^2 = 576 + 49$

$= 625$

$QS = 25$ ins

In $\triangle SQR$

$15^2 + 20^2 = 25^2$

By converse Pythagoras

$\angle R$ is a right $\angle$.

$\therefore$ Quadrilateral is cyclic (opp $\angle$s supplementary)

$\tan \angle PSQ = \frac{7}{24}$ $= 7 \times .04167$

$= .29169 = .2917$

$\tan \angle QSR = \frac{15}{20}$ $\angle PSQ = 16° 16'$

$= .75$

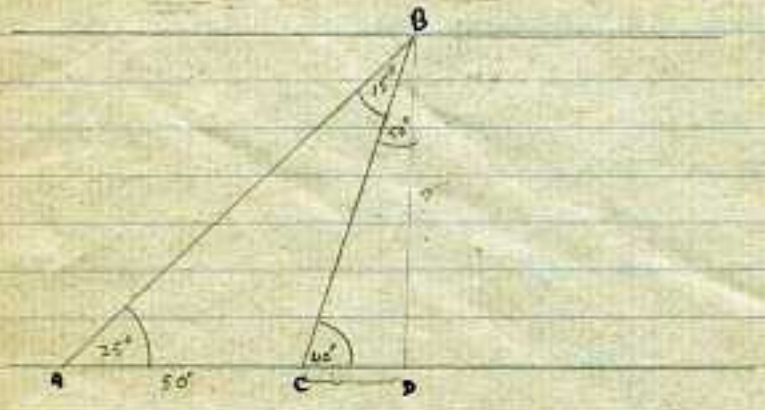
$\angle QSR = 36° 52'$

$\angle S = 53° 8'$

$\angle PSR = 53° 8'$

9

9.



$$BC = \frac{\sin 25^\circ 50}{\sin 15^\circ}$$

$$BD = BC \sin 40^\circ$$

$$BD = \frac{\sin 25^\circ . \sin 40^\circ . 50}{\sin 15^\circ}$$

$$= 52{\cdot}48'$$

$$CD = BD \tan 50^\circ$$

$$= 62{\cdot}55'$$

No	Log.
sin 25°	$\bar{1}.6259$
sin 40°	$\bar{1}.8081$
50	1.6990
	1.1330
sin 15°	$\bar{1}.4130$
52.48	1.7200
tan 50°	0.0762
62.55	1.7962

canal is 52.5 ft wide, man must walk 62.6ft (3 sig figs)

H.J. H. J. Hardy VA. Maths III. March 1955.

6. £ $\frac{2}{9} \times$ 828

= £184 + £210 + £170 = tax free income
= £564

taxable income = £264

tax on first £100 = 300/-
= £15

tax on next £150 = £ $\frac{11}{2} \times \frac{150}{20}$

= £ $\frac{165}{4}$

= £41-5-

tax on rest remainder = 9½ x 14 /-

= £ $\frac{19}{2} \times \frac{14}{20}$

= £ 7½ x $\frac{14}{20}$

= $\frac{15}{2} \times \frac{14}{20}$

= £5 - 5

Total tax = £15 + £46 - 10
= £61 - 10

17

No	$\log_{91}$
14760	4·169
828	2·9180
17·82	1·2511

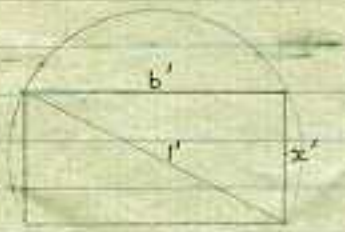
average tax /£ = £ $\frac{61-10}{828}$

= $\frac{1230}{828}$ /-

= $\frac{14760}{828}$ d. $\frac{3890}{207}$ $\frac{14760}{828}$ = 18 d (to nearest 1d)

H.J. Marsh VA. Maths III March 1955.

10.



Let b' be breadth

$l^2 = b^2 + x^2$ by Pythagoras

$b^2 = l^2 - x^2$

$b = \sqrt{(1-x^2)}$

Let s be strength of beam

then $s \propto b.x^3$

$s \propto x^3 b$

$s \propto x^3.\sqrt{(1-x^2)}$

/15 Good answer

Good

Let $y = x^3.\sqrt{(1-x^2)}$

$y^2 = x^5(1-x^2)$

$y^2 = x^5 - x^3$

At a MAX $\frac{dy^2}{dx} = 0$

$\frac{dy^2}{dx} = 5x^4 - 7x^6$

$0 = x^4(5x^2 - 7x^6)$

Let $y = x^3\sqrt{(1-x^2)}$

$y^2 = x^6(1-x^2)$

At a MAX $\frac{dy}{dx} = 0$

$dy/dx = 0$ in other case

$0 = 6x^5 - 8x^7$

$0 = 2x^3(3x^2 - 4x^4)$

either $2x^3 = 0$ or $(3x^2 - 4x^4) = 0$

$x = 0$

or $(3x^2 - 4x^4) = 0$

$x^2(3 - 4x^2) = 0$

either $x = 0$

or $3 - 4x^2 = 0$

MAX or min ?

$3 = 4x^2$

$x^2 = .75$

$x = .866$

$b^2 = l^2 - x^2$

$= 1 - .75$

$b^2 = .25$

$b = .5$

✓ ✓

In strongest beam breadth = 0.5 ft, depth = 0.87 ft (to 2 dec places)