Add Maths 1

370/371/372

UNIVERSITY OF LONDON

General Certificate of Education Examination

JUNE 1973 **ORDINARY LEVEL**

Additional Mathematics 1

PURE MATHEMATICS 1

Two hours

Answer SIX questions. All questions carry equal marks.

All necessary working must be shown.

Candidates are reminded of the necessity for good English and orderly presentation in their answers.

Mathematical formulae and tables are provided.

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1. (i) Solve for x the equations

(*a*) $\log_5 x = 3$,

(*b*) $\log_3 9 + \log_9 x = 2$.

(ii) Given that $\log_5 a + \log_5 b = 4$, find

(*a*) the arithmetic mean of $\log_5 a$, $\log_5 b$,

(*b*) the geometric mean of a, b.

(iii) Find, leaving your answers in logarithmic form, the values of x which satisfy the equation

$$10^{2x} - 10^{x+1} + 16 = 0.$$

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Turn over

(i) Write down the sum and the product of the roots of the equation

$$2x^2 + px + 2 = 0,$$

where p is a constant.

Calculate the two possible values of p if the difference of the roots in this equation is $\frac{3}{2}$.

(ii) The expression $x^3 + ax^2 + bx$ has a factor $(x - 2)$. When the same expression is divided by $(2x + 1)$ the remainder is $\frac{5}{8}$. Find the values of a and b, and sketch the curve

$$y = x^3 + ax^2 + bx$$

when a and b have these values.

(It is not necessary to calculate the coordinates of the turning points.)

From the formula $y = px^2 + qx$, where p and q are constants, values of y were calculated for varying values of x as follows:

x	2	5	7	10	12
y	2	45	77	170	252.

One value of y has been incorrectly printed in this table.

Plot the graph of $\frac{y}{x}$ against x.

Explain why the graph will reveal the incorrect value for y and find

(a) the value of y which is incorrectly printed,

(b) the correct value of y which should be printed,

(c) the numerical values of p and q.

(i) Use the binomial expansion to find the exact value of

$$(2 + \sqrt{3})^4 + (2 - \sqrt{3})^4.$$

Hence deduce two consecutive integers between which the value of $(2 + \sqrt{3})^4$ lies, giving your reasons.

(ii) Calculate all the values of x in the range $0 < x < 360$ which satisfy the equation

$$\sin(180° + x°) = \cos 120°.$$

5. In $\triangle PQR$ the lengths of the sides PQ, QR and RP are 7, 12 and 11 cm respectively. The mid-points of QR and RP are L and M respectively and the altitude from P meets QR at N.
Calculate
(*a*) the cosine of $\angle PQR$,
(*b*) the length of the median PL,
(*c*) the area of $\triangle PQR$,
(*d*) the area of $\triangle MNR$.

(Answers may be given in surd form.)

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6. Calculate the coordinates of the points P and Q, which are equidistant from the points $(-1, 2)$ and $(7, 4)$ and which lie on the bisectors of the angles between the axes.
Calculate
(*a*) the coordinates of R, the remaining vertex of the rectangle $POQR$, where O is the origin of coordinates,
(*b*) the tangent of $\angle ORP$.

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7. Sketch the curve

$$y = \frac{1}{x-2}.$$

Show that the gradient at the point $P\left(4, \frac{1}{2}\right)$ is $-\frac{1}{4}$. The tangent to the curve at P meets the x-axis at T and the line $x = 2$ at Q. Calculate the coordinates of the points T and Q and show that the mid-point of TQ is P.

The normal to the curve at P meets the line $x = 2$ at R. Calculate the coordinates of R and the area of $\triangle QTR$.

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8. (i) Find the area enclosed by the curve $y = x + \frac{1}{x^2}$, the x-axis and the lines $x = 2$ and $x = 5$.

(ii) An open rectangular box with a square base of side x cm is made of thin metal sheeting. The cost of the sheeting used for the base is 3p per cm² and of that used for the sides 1p per cm². The total cost of the sheeting is 225p. Show that the volume, V cm³, of the box is given by $V = \frac{1}{4}x(225 - 3x^2)$. Hence find the maximum volume of the box.

$(x-2)^{-1/2}$ $= -\frac{1}{2}(x - \ldots)$ 1
$-\frac{x}{2} + 1$