Pure Maths 1
Maths (P and A) 1
Pure Maths with Stats 1

380/400/420

UNIVERSITY OF LONDON

General Certificate of Education Examination

JUNE 1974 **ADVANCED LEVEL**

Mathematics

PURE MATHEMATICS 1

MATHEMATICS (PURE AND APPLIED) 1

PURE MATHEMATICS WITH STATISTICS 1

Three hours

Full marks may be obtained for answers to EIGHT questions.

If more than eight questions are attempted, only the BEST EIGHT answers will be taken into account.

All questions carry equal marks.

Mathematical formulae and tables are provided.

TSE 73/676 15/3/3/100/35000

Turn over

1. (i) If $ax^2 + bx + c \equiv a\{(x+p)^2 + q\}$, $(a \neq 0)$, express p and q in terms of a, b and c. Deduce that, if $b^2 < 4ac$, the expression has the same sign as a for all real values of x.

If

$$g(x) = (k-6) + (k-3)x - x^2,$$

find the set of values of k for which $g(x)$ is always negative.

(ii) If $f(r) = \log\left(1 + \dfrac{1}{r}\right)$, show that

$$f(1) + f(2) + \ldots + f(n) = f\left(\frac{1}{n}\right).$$

---

2. (i) Express each of the complex numbers

$$z_1 = (1-i)(1+2i), \quad z_2 = \frac{2+6i}{3-i}, \quad z_3 = \frac{-4i}{1-i}$$

in the form $a + bi$, where a and b are real.

Show that $|z_2 - z_1| = |z_1 - z_3|$ and that, for principal values of the arguments, $\arg(z_2 - z_1) - \arg(z_1 - z_3) = \pi/2$. If z_1, z_2, z_3 are represented by points P_1, P_2, P_3 respectively in an Argand diagram, prove that P_1 lies on the circle with P_2P_3 as diameter.

(ii) Using de Moivre's Theorem, or otherwise, find an expression for $\cos 6\theta$ in ascending powers of $\cos\theta$.

---

3. (i) Nine counters are identical except for their colour. Two are blue, two red, two green, two yellow and one is black. How many distinguishable sets of three counters can be selected from the nine?

(ii) Prove by induction, or otherwise, that

$$1.1! + 2.2! + 3.3! + \ldots + n.n! = (n+1)! - 1.$$

---

4. (i) Expand $(1 - x^2)^{1/2}$ as a series of ascending powers of x as far as the term in x^6. By putting $x = 0{\cdot}1$, obtain an approximation to $\sqrt{11}$ to 5 places of decimals.

(ii) The table shows approximate values of a variable y corresponding to certain values of another variable x. By drawing a suitable linear graph, verify that these values of x and y satisfy approximately a relationship of the form $y = ax^k$. Use your graph to find approximate values of the constants a and k.

x	5	10	15	20	25	30
y	45	63	77	89	100	110

---

5. If P, Q are the points (x_1, y_1), (x_2, y_2), show that the equation of the circle on PQ as diameter is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0\,.$$

If the tangents from the origin O touch the circle $x^2 + y^2 - 8x - 4y + 10 = 0$ at A and B, find the equation of the circle OAB and the equation of the line AB.

---

6. Prove that the equation of the chord joining the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ of the parabola $y^2 = 4ax$ is

$$2x - (p + q)y + 2apq = 0\,.$$

S is the focus of the parabola and M is the mid-point of PQ. The line through S perpendicular to PQ meets the directrix at R.

Prove that

$$2\,RM = SP + SQ\,.$$
